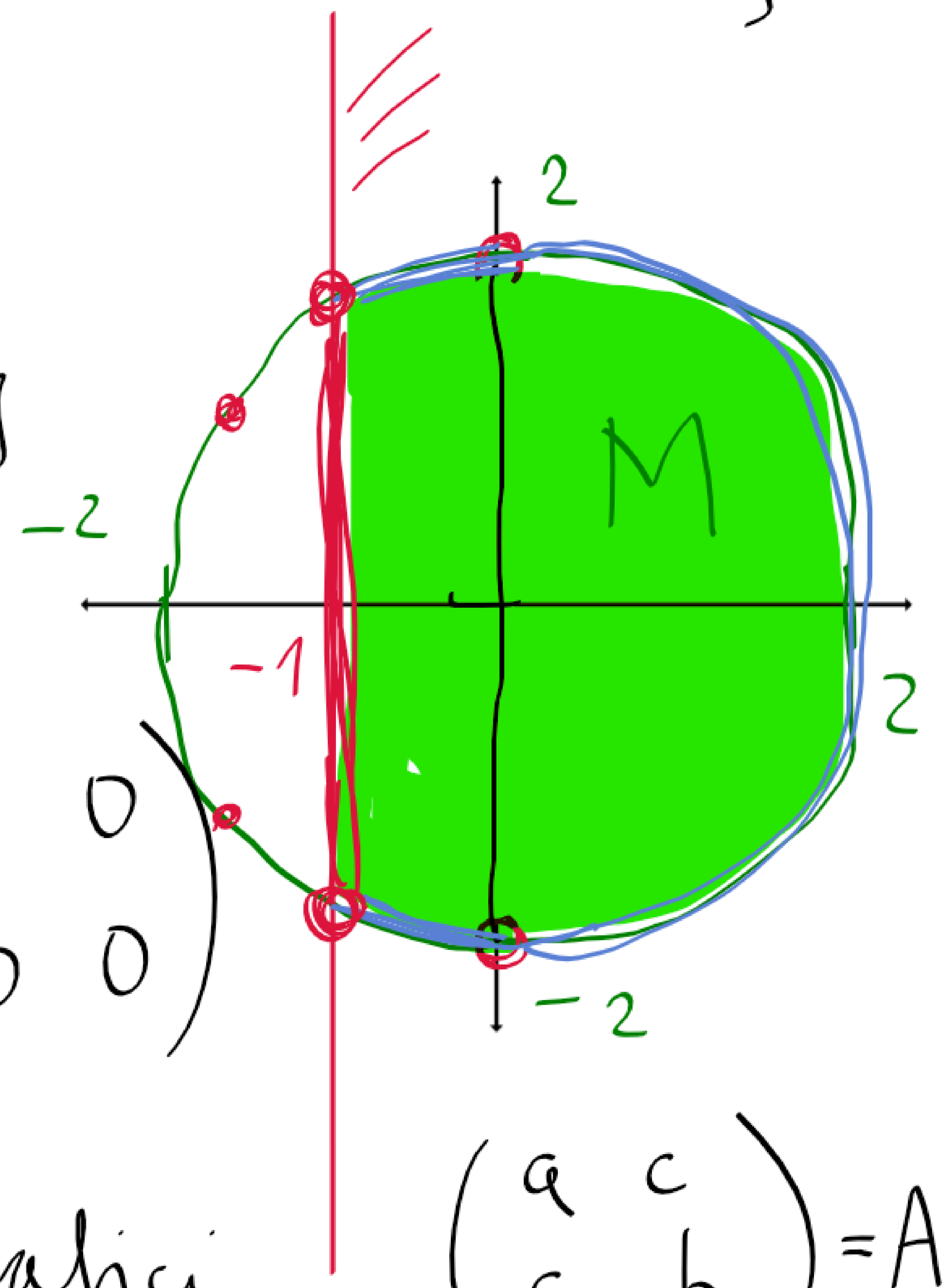
13/4/a) $f(x,y) = x^4 y \qquad f_x = 4x^3 y \qquad f_y = x^4$

$M = \{(x,y)\in\mathbb{R}^2 : x^4+y^4 \le 16 \ \wedge \ x \ge -1\}$

S.B.: $x = 0$, $y \in \mathbb{R}$.

P.B.: $\{(x,y) : x = 0 \quad (x,y)\in M^\circ\}$
$= \{0\}\times(-2,2)$



$$d^2 f(x,y) = \begin{pmatrix} 12x^2 y & 4x^3 \\ 4x^3 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

Sylvestrovo pravidlo: Pro matici $\begin{pmatrix} a & c \\ c & b \end{pmatrix} = A$

A je PD $\iff$ $a > 0 \ \wedge \ \det A > 0$

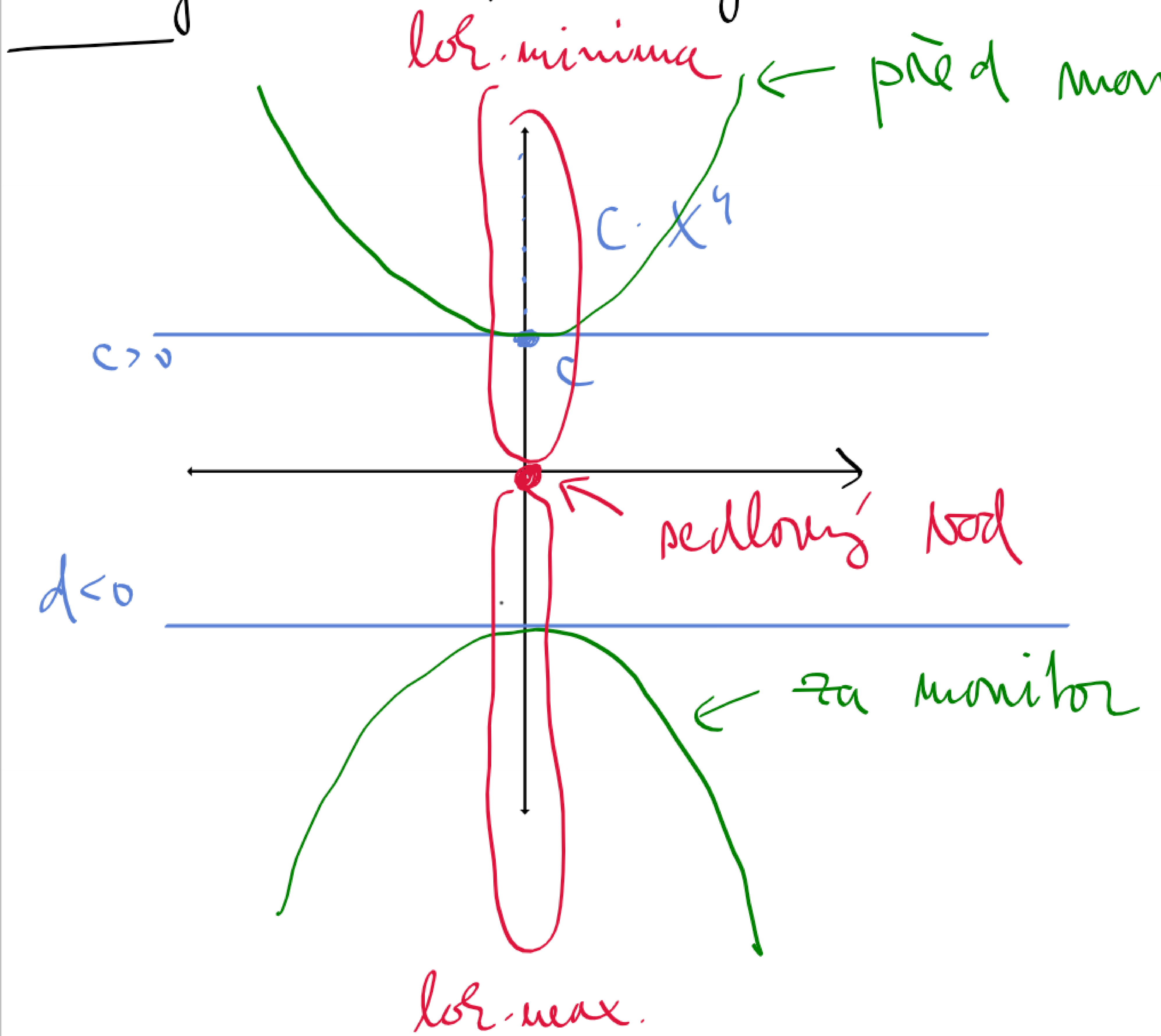
ND $\quad a < 0 \ \wedge \ \det A > 0$

PSD $\iff a \ge 0 \ \wedge \ \det A \ge 0 \ \wedge \ b \ge 0$

NSD $\iff a \le 0 \ \wedge \ \det A \ge 0 \ \wedge \ b \le 0$

$$\det\left(d^2 f(x,y)\right) = -4x^3\cdot 4x^3 = -16x^6 \le 0$$

Nezjistíme nic vyšetřováním Hessovy mce.



$x^4+y^4 = 16, \ x = -1$

$y^4 = 15$

$y = \pm\sqrt[4]{15}$

Chování na hranici M:

$$H(M) = \{\cancel{0}\ -1\}\times \cancel{(-2,2)}\ \left(-\sqrt[4]{15}, \sqrt[4]{15}\right) \cup \{(0,-2),(0,2)\} \ \cup$$

$$\cup \{(x,y)\in\mathbb{R}^2 : x^4+y^4 = 16, \ \boxed{x > -1}\}$$

• $f(x,y) = x^4 y$ $\qquad M_1 := \{-1\} \times (-\sqrt[4]{15}, \sqrt[4]{15})$

$g(y) := f(0,y)$ ... extrémy přes

$= (-1)^4 \cdot y = y$ ... nemá extrémy na $(-\sqrt[4]{15}, \sqrt[4]{15})$.

• $M_2 = \{(x,y) \in \mathbb{R}^2 ;\ x^4 + y^4 = 16 \wedge x > -1\}$

$g(x,y) = x^4 + y^4 - 16$ $\qquad M_2 \subseteq g^{-1}(\{0\})$.

$\nabla f = -\lambda \nabla g$

$[\nabla f + \lambda \nabla g = 0] \Leftrightarrow [\text{Tj. } f_x + \lambda g_x = 0 \ \wedge\ f_y + \lambda g_y = 0]$

(1) $x^4 + y^4 = 16$

(2) $4x^3 y + \lambda \cdot 4x^3 = 0$

(3) $x^4 + \lambda 4y^3 = 0$

Řešení soustavy: (2) $\Leftrightarrow$ $x^3(y+\lambda) = 0$

$\Rightarrow$ $x = 0$ $\vee$ $y = -\lambda$.

$x = 0$: (1) $0^4 + y^4 = 16 \Leftrightarrow y = \pm 2$

$[0,-2]$, $[0,2]$

$y = -\lambda$: (3) $x^4 + \lambda \cdot 4(-\lambda)^3 = 0$

Tj. $x^4 = 4\lambda^4$

(1) $4\lambda^4 + \lambda^4 = 16$ $\quad$ tj. $\lambda^4 = \frac{16}{5}$,

tj. $\lambda = \pm \sqrt[4]{\frac{16}{5}} = \pm \frac{2}{\sqrt[4]{5}}$

$\lambda = \frac{2}{\sqrt[4]{5}} \Rightarrow x^4 = 4 \cdot \frac{16}{5} = \frac{64}{5}$

$x = \pm \sqrt[4]{\frac{64}{5}}$

$\left[\sqrt[4]{\frac{64}{5}}, \frac{-2}{\sqrt[4]{5}}\right]$, $\cancel{\left[-\sqrt[4]{\frac{64}{5}}, \frac{-2}{\sqrt[4]{5}}\right]}$ $\quad < -1$

$\lambda = -\frac{2}{\sqrt[4]{5}} \Rightarrow$

$\left[\sqrt[4]{\frac{64}{5}}, \frac{2}{\sqrt[4]{5}}\right]$, $\cancel{\left[-\sqrt[4]{\frac{64}{5}}, \frac{2}{\sqrt[4]{5}}\right]}$ $\quad < -1$

• $[-1, -\sqrt[4]{15}]$, $[-1, \sqrt[4]{15}]$

TJ. 6 ~~ℓ~~ PB

Stačí dosadit

(1) $x^2+y^2+z^2=1$

(2) $1+\lambda 2x=0 \implies x=\dots$

(3) $-2+\lambda 2y=0 \quad y=\dots$

(4) $-2+\lambda 2z=0 \quad z=\dots$

2b) (1) $x^2+y^2=1$ $\qquad f(x,y)=x^2+y$

(2) $2x+\lambda\cdot 2x=0$

(3) $1+\lambda\cdot 2y=0$

(2) $\iff x(1+\lambda)=0 \iff \underline{x=0} \vee \underline{\lambda=-1}$

Dvě větve řešení:

(i) $\underline{x=0}$; ...

(ii) $\underline{\lambda=-1}$ ...

$\Rightarrow$ PB

2d) $f(x,y,z)=x-y+3z$

(1) $x^2+y^2+4z^2=4$

(2) $1+\lambda\cdot 2x=0$

(3) $-1+\lambda\cdot 2y=0$ $\qquad$ podobně jako dříve

(4) $3+\lambda\cdot 8z=0$

2g)

12/2/c) $f(x,y,z) = x^3 + y^2 + z^2/2 - 3xz - 2y + 2z$

Najděte lok. extrémy: 2 KROKY

1) PB: SB

2) Hessova matice PD → min. apod.

$f_x = 3x^2 - 3z = 0$

$f_y = 2y - 2 = 0 \iff y = 1$

$f_z = z - 3x + 2 = 0 \Rightarrow z = 3x - 2$

Tedy $3x^2 - 3(3x-2) = 0$

$x^2 - 3x + 2 = 0$

$(x-1)(x-2) = 0 \qquad x = 1 \quad \vee \quad x = 2$

$z = 1 \qquad z = 4$

P.B: $[1,1,1]$, $[2,1,4]$

$$d^2 f(x,y) = \begin{pmatrix} f_{xx} & f_{yx} & f_{zx} \\ f_{xy} & f_{yy} & f_{zy} \\ f_{xz} & f_{yz} & f_{zz} \end{pmatrix} =$$

$$= \begin{pmatrix} 6x & 0 & -3 \\ 0 & 2 & 0 \\ -3 & 0 & 1 \end{pmatrix}$$

$2\cdot(3) - (1)$

$[1,1,1]$: $\begin{pmatrix} 6 & 0 & -3 \\ 0 & 2 & 0 \\ -3 & 0 & 1 \end{pmatrix} \sim \begin{pmatrix} 6 & 0 & -3 \\ 0 & 2 & 0 \\ 0 & 0 & -1 \end{pmatrix} \sim \begin{pmatrix} 6 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -1 \end{pmatrix}$

Matice ID, neboť má na diagonále kladná i záporná čísla.

Tedy $[1,1,1]$ není bodem extrému.

$$\begin{pmatrix} 6x & 0 & -3 \\ 0 & 2 & 0 \\ -3 & 0 & 1 \end{pmatrix}$$

$$>0 \qquad >0$$

$$[2,1,4]: \quad \begin{pmatrix} 12 & 0 & -3 \\ 0 & 2 & 0 \\ -3 & 0 & 1 \end{pmatrix} =: A$$

$$\det A = 12 \cdot 2 \cdot 1 - (-3) \cdot 2 \cdot (-3) =$$

$$= 24 - 18 = 6 > 0$$

Tedy $d^2 f(2,1,4)$ je PD podle Sylvestrova př. $\Rightarrow$ $[2,1,4]$ je bod lokálního min.